Fisher, R.A.
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into 5 independent sets of 3 each, being the 3 independent comparisons among 4 sets of 4 objects each, into which the whole may be divided. An arrangement of this kind is shown below, in which numeral suffices are used in place of one set of Greek letters :—

$A_1\alpha$	$B_2\beta$	$C_3\gamma$	$D_4\delta$
$B_4\gamma$	$A_3\delta$	$D_2\alpha$	$C_1\beta$
$C_2\delta$	$D_1\gamma$	$A_4\beta$	$B_3\alpha$
$D_3\beta$	$C_4\alpha$	$B_1\delta$	$A_2\gamma$

There are in all $2\times6\times24^3$ such arrangements.

The 5×5 Latin squares in the standard position are 56 in number, and fall into two sets. One set of 50 yields no Græco-Latin square, but the set of 6, which are symmetrical about the diagonal, yield each 3 different squares which do not differ merely in a permutation of the Greek letters. There are therefore $3\times6\times24\times120^2$ 5×5 Græco-Latin squares. The three different arrangements are all mutually orthogonal, so that we may add a numeral suffix, as in the 4×4 square above, and obtain $6\times6\times24\times120^3$ solutions. And we may add a second suffix independent of the first, and of the letters, in $6\times6\times24\times120^4$ different ways. An example using two suffices is shown below :—

$A_1\alpha_1$	$B_2\beta_2$	$C_3\gamma_3$	$D_4\delta_4$	$E_5\epsilon_5$
$B_3\delta_5$	$C_4\epsilon_1$	$D_5\alpha_2$	$E_1\beta_3$	$A_2\gamma_4$
$C_5\beta_4$	$D_1\gamma_5$	$E_2\delta_1$	$A_3\epsilon_2$	$B_4\alpha_3$
$D_2\epsilon_3$	$E_3\alpha_4$	$A_4\beta_5$	$B_5\gamma_1$	$C_1\delta_2$
$E_4\gamma_2$	$A_5\delta_3$	$B_1\epsilon_4$	$C_2\alpha_5$	$D_3\beta_1$

Consequently, the 24 degrees of freedom among 25 objects can be subdivided into 6 independent sets of 4 corresponding to the rows, columns, Latin letters, first suffices, Greek letters and second suffices in the square above. Such a square may be said to be completely orthogonal.

Completely Orthogonal 8×8 *Square*

1 1 1 1 1 1 1	2 5 7 3 8 4 6	3 2 5 4 7 6 8	4 3 2 6 5 8 7	5 7 8 2 6 3 4	6 4 3 8 2 7 5	7 8 6 5 4 2 3	8 6 4 7 3 5 2
2 2 2 2 2 2 2	1 8 3 7 5 6 4	7 1 8 6 3 4 5	6 7 1 4 8 5 3	8 3 5 1 4 7 6	4 6 7 5 1 3 8	3 5 4 8 6 1 7	5 4 6 3 7 8 1
3 3 3 3 3 3 3	7 4 2 1 6 5 8	1 7 4 5 2 8 6	5 1 7 8 4 6 2	4 2 6 7 8 1 5	8 5 1 6 7 2 4	2 6 8 4 5 7 1	6 8 5 2 1 4 7
4 4 4 4 4 4 4	6 3 8 5 7 1 2	5 6 3 1 8 2 7	1 5 6 2 3 7 8	3 8 7 6 2 5 1	2 1 5 7 6 8 3	8 7 2 3 1 6 5	7 2 1 8 5 3 6
5 5 5 5 5 5 5	8 1 6 4 2 3 7	4 8 1 3 6 7 2	3 4 8 7 1 2 6	1 6 2 8 7 4 3	7 3 4 2 8 6 1	6 2 7 1 3 8 4	2 7 3 6 4 1 8
6 6 6 6 6 6 6	4 7 5 8 3 2 1	8 4 7 2 5 1 3	2 8 4 1 7 3 5	7 5 3 4 1 8 2	1 2 8 3 4 5 7	5 3 1 7 2 4 8	3 1 2 5 8 7 4
7 7 7 7 7 7 7	3 6 1 2 4 8 5	2 3 6 8 1 5 4	8 2 3 5 6 4 1	6 1 4 3 5 2 8	5 8 2 4 3 1 6	1 4 5 6 8 3 2	4 5 8 1 2 6 3
8 8 8 8 8 8 8	5 2 4 6 1 7 3	6 5 2 7 4 3 1	7 6 5 3 2 1 4	2 4 1 5 3 6 7	3 7 6 1 5 4 2	4 1 3 2 7 5 6	1 3 7 4 6 2 5

Although there are 9408 6×6 Latin squares in the standard position, belonging to 12 distinct types, yet none of these yields a Græco-Latin square, a conclusion arrived at by Euler after a considerable investigation, but only recently established for certain by the enumeration of the actual types which occur. Græco-Latin squares are easily formed with 7 units in a side, or any other odd number; the 7×7 squares were enumerated by Norton in 146 species; later A. Sade found the 147th species, adding 14,112 standard squares to the 16,927,968 found by Norton. It may be shown that with any prime number (p) the p^2-1 degrees of freedom among p^2 objects may be separated into $p+1$

independent sets of $p-1$ degrees of freedom each, each representing comparisons among p groups of p objects. Yates has shown that this is also true of 8^2 objects, and the same can be done with 9^2, as the examples illustrate. Stevens has demonstrated the possibility in general for powers of prime numbers.

After the rows and columns, the categories of subdivision are represented by the numbers in the cells of each square. The squares given may be randomised by permuting, (*a*) the rows, columns, and each set of numbers among themselves; (*b*) whole sets of numbers with each other and with the rows and columns.

Thus, in the 8×8 square shown above, the seven numbers represent seven different ways of dividing 64 objects into 8 groups of 8 each, making with the rows and columns 9 ways in all, so that no two objects are classified alike in any two of the nine ways.

Completely Orthogonal 9×9 *Square*

1111	2322	3233	9549	7457	8668	5975	6886	4794
1111	4546	7978	3834	6369	9492	2627	5753	8285
2246	3154	1365	7672	8583	9491	6717	4928	5839
8322	2454	5889	7715	1247	4673	9538	3961	6196
3378	1289	2197	8414	9625	7536	4843	5751	6962
6233	9665	3797	5926	8158	2581	4419	7842	1374
4435	5616	6524	3861	1742	2953	8399	9277	7188
9744	3279	6612	8567	2993	5135	7351	1486	4828
5567	6448	4659	1996	2874	3785	9132	7313	8221
4955	7187	1523	6448	9871	3316	5262	8694	2739
6693	4571	5482	2738	3919	1827	7264	8145	9356
2866	5398	8431	1659	4782	7224	3143	6575	9917
7729	8937	9818	6255	4166	5344	2681	3592	1473
5477	8813	2345	4291	7636	1768	6984	9129	3552
8852	9763	7941	4387	5298	6179	3426	1634	2515
3688	6721	9256	2172	5514	8949	1895	4337	7463
9984	7895	8776	5123	6331	4212	1558	2469	3647
7599	1932	4164	9383	3425	6857	8776	2218	5641

35·01 Configurations

Instead of consideri
a configuration of n^3 e
n^2 elements each, there
intersecting at right an
different sets will interse

The first n letters
assigned to n^2 elements
each layer of each of th
cube. If the process v
the Greek alphabet, w
each of the Latin letters
the Greek letters, we sh
The greatest possible n
be accommodated in
n^2+n-2, but so many
particular values of n.
of subdivision of the e
provided by the three
maximum number of m

An elementary pro
somewhat an intricate o

Let us suppose tha
dividing the n^3 elemen
each. Then the numbe
respect to any one categ

$\frac{1}{2}$

and for all categories th

Now any pair of ele
to a certain number of