

Matrix Algebra Review: Self Assessment Exam

1 Matrix Definitions (1 point each)

1.1) What is the dimension of the matrix, A, where

$$A = \begin{pmatrix} 8 & -5 \\ 2 & -1 \\ 2 & 5 \end{pmatrix}$$

1.2) What is the value of the element in the third row and second column of A, $a_{3,2}$?

1.3) What is the value of $a_{1,1}$?

1.4) What is the dimension of the matrix, B, where

$$B = \begin{pmatrix} 1 & -5 & 8 & 0 \\ 2 & 0 & 8 & 4 \\ -3 & 6 & 8 & 2 \\ 2 & 5 & 7 & -5 \end{pmatrix}$$

1.5) What is the value of $b_{2,1}$?

1.6) What is the value of $b_{3,3}$?

1.7) Let $c = 12.4$; is c a matrix, vector, or scalar?

1.8) Let $d = (-1 \ 5 \ 4 \ 0)$; is d a matrix, vector, or scalar?

2 Matrix Arithmetic (1 point each)

Use the following matrices and vectors for problems in this section:

$$A = \begin{pmatrix} 0 & -8 & 5 & 1 & -4 \\ 7 & 0 & -3 & -10 & 3 \\ 5 & 5 & -15 & -1 & -6 \end{pmatrix}, \quad B = \begin{pmatrix} 12 & -3 & 3 \\ 1 & -3 & -2 \\ -8 & -2 & -1 \\ 12 & 0 & -6 \\ -5 & -2 & 3 \end{pmatrix}$$

$$C = \begin{pmatrix} -4 & 2 & -10 \\ -5 & 6 & 2 \\ 7 & -1 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & 1 & 3 \\ 1 & -7 & -5 \\ 9 & 0 & -6 \end{pmatrix}, \quad E = \begin{pmatrix} 9 & 7 & 1 \\ 8 & -8 & 8 \\ 1 & -4 & -1 \end{pmatrix}$$

$$f = (6 \quad -8 \quad -4 \quad -4 \quad -6), \quad g = \begin{pmatrix} 2 \\ 0 \\ -5 \\ 7 \\ -8 \end{pmatrix}$$

2.1) Find A^T . Is A a symmetric matrix?

2.1) Find C^T . Is C a symmetric matrix?

2.3) Calculate $C + D$

2.4) Calculate $E + D$

2.5) Calculate $2 * (C - D) + 4 * E$

2.6) Calculate $A * g$

2.7) Calculate $f \cdot g$

2.8) Calculate $2 * f * B$

2.9) Calculate $A * B + 3 * C$

2.10) Calculate $C * (D + E)$

3 Matrix Properties (1 point each)

3.1) Let $A = \begin{pmatrix} -3 & 3 \\ 5 & -5 \end{pmatrix}$, $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Find the trace of A.

3.2) Find the determinate of A.

3.3) Find the trace of $-2 * A * I_2$

3.4) Let $B = \begin{pmatrix} -4 & 2 & -10 \\ -5 & 6 & 2 \\ 7 & -1 & -1 \end{pmatrix}$, $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find the trace of $B + I_3$.

3.5) Find the determinate of B.

3.6) Find the determinate of $4 * B$

3.7) Let $C = \begin{pmatrix} -1 & 7 & 12 & 6 \\ 2 & -2 & -2 & -3 \\ 5 & 9 & -4 & 5 \\ -6 & 6 & 2 & -5 \end{pmatrix}$. Find the determinate of C.

3.8) Find the determinate of C^T

4 Matrix Inverse (1 point each)

4.1) Let $A = \begin{pmatrix} -3 & 2 \\ 5 & -5 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 \\ -1 & -5 \end{pmatrix}$. Are A and B inverses of each other?

4.2) Let $C = \begin{pmatrix} -1 & -0.4 \\ -1 & -0.6 \end{pmatrix}$. Are A and C inverses of each other?

4.3) Let $D = \begin{pmatrix} -3 & 3 \\ 5 & -5 \end{pmatrix}$. Is D invertible? If so, find D^{-1} .

4.4) Let $E = \begin{pmatrix} 7 & 0 \\ -2 & -5 \end{pmatrix}$. Is E invertible? If so, find E^{-1} .

5 Advanced Topics (1 point each)

5.1) Let $a = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$, $c = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$. Are vectors a and b orthogonal? Are vectors a and c orthogonal?

5.2) Are vectors b and c linearly independent? Are vectors a , b , and c linearly independent?

5.3) Find $\|a\|_2$ and $\|b\|_\infty$

5.4) Let $D = \begin{pmatrix} 2 & 7 & -9 \\ 0 & 7 & -2 \\ -10 & -1 & 8 \end{pmatrix}$. Find $\|D\|_F$.

5.5) Find the rank of D.

5.6) Find the nullity of D.

5.7) Is $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ in reduced-row echelon form? Is $F = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ in reduced-row echelon form?

5.8) Let $G = \begin{pmatrix} 3 & -2 & 0 \\ 5 & 3 & 1 \\ -5 & 4 & -2 \end{pmatrix}$. Find the inverse of G using Gauss-Jordan Elimination.

5.9) Let $H = \begin{pmatrix} 5 & -2 \\ 0 & -4 \end{pmatrix}$. Find the eigenvalues of H and the corresponding eigenvectors.

5.10) Let $I = \begin{pmatrix} -1 & -2 \\ -2 & 4 \end{pmatrix}$. Find the eigenvalues of I and the corresponding eigenvectors.