

Matrix Algebra Review: Self Assessment Solutions

1 Solutions to Matrix Definitions Problems

1.1) 3×2

1.2) $a_{3,2} = 5$?

1.3) What is the value of $a_{1,1} = 8$?

1.4) 4×4

1.5) $b_{2,1} = 2$

1.6) What is the value of $a_{3,3} = 8$

1.7) c is a scalar

1.8) d is a vector

2 Solutions to Matrix Arithmetic Problems

2.1) A is not symmetric.

$$A^T = \begin{pmatrix} 0 & 7 & 5 \\ -8 & 0 & 5 \\ 5 & -3 & -15 \\ 1 & -10 & -1 \\ -4 & 3 & -6 \end{pmatrix}$$

2.2) C is not symmetric.

$$C^T = \begin{pmatrix} -4 & -5 & 7 \\ 2 & 6 & -1 \\ -10 & 2 & -1 \end{pmatrix}$$

2.3) $C + D = \begin{pmatrix} 0 & 3 & -7 \\ -4 & -1 & -3 \\ 16 & -1 & -7 \end{pmatrix}$

2.4) $E + D = \begin{pmatrix} 13 & 8 & 4 \\ 9 & -15 & 3 \\ 10 & -4 & -7 \end{pmatrix}$

2.5) $2 * (C - D) + 4 * E = \begin{pmatrix} 20 & 30 & -22 \\ 20 & -6 & 46 \\ 0 & -18 & 6 \end{pmatrix}$

2.6) $A * g = \begin{pmatrix} 14 \\ -65 \\ 126 \end{pmatrix}$

$$2.7) f * g = 52$$

$$2.8) 2 * f * B = (156 \quad 52 \quad 88)$$

$$2.9) A * B + 3 * C = \begin{pmatrix} -28 & 28 & -37 \\ -42 & -3 & 99 \\ 224 & 9 & 5 \end{pmatrix}$$

$$2.10) C * (D + E) = \begin{pmatrix} -134 & -22 & 60 \\ 9 & -138 & -16 \\ 72 & 75 & 32 \end{pmatrix}$$

3 Solutions to Matrix Properties Problems

$$3.1) \text{tr}(A) = -8$$

$$3.2) \det(A) = 0$$

$$3.3) \text{tr}(-2 * A * I_2) = 16$$

$$3.4) \text{tr}(B + I_3) = 4$$

$$3.5) \det(B) = 404$$

$$3.6) \det(4 * B) = 25856$$

$$3.7) \det(C) = -4240$$

$$3.8) \det(C^T) = -4240$$

4 Solutions to Matrix Inverse Problems

4.1) A and B are not inverses of each other because

$$AB = \begin{pmatrix} -23 & -19 \\ 40 & 40 \end{pmatrix}$$

4.2) A and C are inverses of each other because

$$AC = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

4.3) D is not invertible because $\det(D) = 0$

4.4) E is invertible because $\det(E) = -35 \neq 0$.

$$E^{-1} = \begin{pmatrix} \frac{1}{7} & 0 \\ \frac{2}{35} & -0.2 \end{pmatrix}$$

5 Solutions to Advanced Topics Problems (1 point each)

5.1) Vectors a and b are not orthogonal. Vectors a and c are orthogonal.

5.2) Vectors b and c are linearly independent. Vectors a , b , and c are also linearly independent.

5.3) $\|a\|_2 = \sqrt{24} = 4.898979$ and $\|b\|_\infty = 3$

5.4) $\|D\|_F = 18.76$

5.5) $\text{rank}(D) = 3$.

5.6) $\text{nullity}(D) = 0$.

5.7) E is not in reduced-row echelon form because it violates condition three. F is in reduced-row echelon form.

$$5.8) G^{-1} = \begin{pmatrix} 0.250 & 0.10 & 0.050 \\ -0.125 & 0.15 & 0.075 \\ -0.875 & 0.05 & -0.475 \end{pmatrix}$$

5.9) The eigenvalues of H are 5 and -4 and they have corresponding eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2/9 \\ 9 \end{pmatrix}$ respectively.

5.10) The eigenvalues of I are $\frac{1}{2}(3 - \sqrt{41})$ and $\frac{1}{2}(3 + \sqrt{41})$ and they have corresponding eigenvectors $\begin{pmatrix} \frac{1}{4}(5 - \sqrt{41}) \\ 1 \end{pmatrix}$ and $\begin{pmatrix} \frac{1}{4}(5 + \sqrt{41}) \\ 1 \end{pmatrix}$ respectively.